The Curious Quest

Issue Number 0

Centre for Mathematical Outreach

"The journey of a thousand miles begins with a single step"

– Lao Tzu

§ Reader's Delight

Sophie Germain

In the late 18th century, young Sophie Germain fell in love with mathematics. But there was a problem—universities didn't accept women. Determined, she secretly obtained lecture notes from the prestigious École Polytechnique and submitted assignments under the pseudonym "Monsieur LeBlanc."

Her work in Number Theory was so impressive that it caught the attention of the great Carl Friedrich Gauss. When Gauss discovered that "LeBlanc" was actually a woman, he was stunned and wrote her a heartfelt letter, praising her genius and perseverance.

Germain later contributed significantly to Fermat's Last Theorem, earning a special class of prime numbers named after her—Sophie Germain primes. She proved that talent and determination could break barriers, even in a world that tried to keep her out!

Figure 1: Sophie Germain

Bell, E. T. (1937). **Men of Mathematics.** Simon & Schuster.

§ The Problem Arena

Problem 1

Find the remainder when $(x+1)^n$ is divided by x^3+x^2+x+1.

Problem 2

Find the number of isosceles triangles with integer sides if no side exceeds 2026.

Problem 3

Let $\mathbb{L}$ denote the set of lattice points in $\mathbb{R}^2$, that is, the set of all points $(x, y) : x, y \in \mathbb{Z}$. And define $d : \mathbb{L} \times \mathbb{L} \to \mathbb{R}$, $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ where $x \equiv (x_1, x_2)$, $y \equiv (y_1, y_2)$. Prove that $(\mathbb{L}, d)$ is a metric space. Is the shortest path in this metric space unique? What is the relation between this metric and the standard euclidean metric? State the cases for equality.

§ The Enigma Box

A Weird Power Tower

Nolan Grayson came across the following problem in his VBSE (Viltrum Board of Secondary Education) exam:
If $x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}} = 4$ find the value of x.
So he began solving this, noticing the key to this problem was realizing that we can just substitute 4 in the power, that is, $x^4 = 4 \Rightarrow x = \sqrt[4]{4} = \sqrt{2}$, VBSE intended this to be the answer, but our curious Nolan thought that if the question had instead asked him to find the value of x for $x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}} = 2$, he'd get $x = \sqrt{2}$! Since Nolan is in 10th grade, he has not yet encountered the formal study of sequences (hint?), but you have! So help him challenge the VBSE by writing up a rigorous argument, arguing what the real value of $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}$ is.

Figure 2: Nolan Grayson

§ This Issue's Contributors

- Question 1 : *Aadi Upraity, St. Xavier's College.*
- Question 2 : *Aadi Upraity, St. Xavier's College.*
- Question 3 : *Aadi Upraity, St. Xavier's College.*
- POTW : *Aadi Upraity, St. Xavier's College.*
- Reader's Delight : *Harsh Nagda, St. Xavier's College.*

Check out the next issue for hints and solutions!

If you have any questions, puzzles or stories that you want to share then kindly mail them to centre.math.outreach@gmail.com!