The Curious Quest

Issue Number 4

Centre for Mathematical Outreach, MAX

"A mathematician is a machine for turning coffee into theorems."

– Alfred Renyi

§ Reader's Delight

The mathematician that never existed

Figure 1: Bourbaki council

Nicolas Bourbaki is likely the last mathematician to master nearly all aspects of the field. A consummate collaborator, he made fundamental contributions to important mathematical fields such as set theory and functional analysis. He also revolutionized mathematics by emphasizing rigor in place of conjecture. There's just one problem: Nicolas Bourbaki never existed.

The story of Bourbaki's Secret Identity is one of the most famous pranks in mathematical history. In the 1930s, a group of young French mathematicians—including André Weil, Henri Cartan, and Jean Dieudonné—grew frustrated with the disorganized state of mathematical textbooks. They decided to rewrite mathematics in a rigorous and unified way under the pseudonym Nicolas Bourbaki, presenting him as a real but eccentric professor.

The deception was so successful that universities received applications from "Bourbaki" for teaching positions, and even established mathematicians tried to meet him. The group's influence reshaped modern mathematics, particularly in abstract algebra and topology. Their style was extremely formal, often eliminating intuition in favor of strict axiomatic structures. The prank eventually became public knowledge, but by then, "Bourbaki" had already left a lasting legacy on mathematics.

The Conversation. (2019, December 27). *Genius mathematician who never existed: Nicolas Bourbaki.* Sci.News.

§ The Problem Arena

Problem 1

Define the sequence $\{S_n\}$ for all positive integers as follows:

$$S_n = \ln(\sqrt[n^2]{1 \cdot 2 \cdots n^n}) - \ln\sqrt{n},$$

Find $\lim_{n\to\infty} S_n$

Problem 2

Let $X_1, X_2, ..., X_k$ be nonnegative integers, Compute:

$$\lim_{n\to\infty}\left(\sum_{i=1}^{k} X_i^n\right)^{\frac{1}{n}}$$

Problem 3

Define the subspace V of $\mathbb{C}[x]$ as:

$$V := \{f \in \mathbb{C}[x] : \deg(f) \leq 50 \text{ and } f(iz) = -f(z)\ \forall z \in \mathbb{C}\}$$

Find $\dim V$.

§ The Enigma Box

Doom Invariance

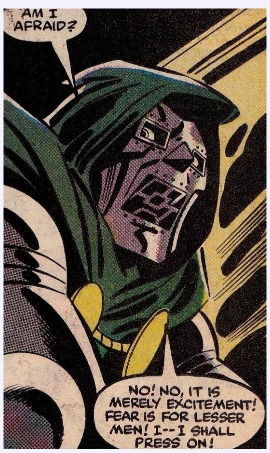

Figure 2: Dr. Doom

After visiting many universes, Dr. Doom has 2025 Infinity Stones. He splits them into two groups and harnesses energy from the interaction between the two by multiplying the number of stones in each group. For example, if he splits them into 1000 and 1025 stones, he gains 10,25,000 units of energy.
But Doom isn't done by splitting them only once. He keeps splitting each group further—again and again, always multiplying the sizes of the two subgroups he creates and adding that product to his total energy, which means that let's say he splits the stones into x and y units in the first turn, then splits the x pile into x_1 and x_2 and y pile into y_1 and y_2, then the total amount of energy gained by him is $xy + x_1x_2 + y_1y_2$. He stops once all the piles have only one stone each (these give no energy on their own).
No matter how Doom chooses to split the stones at each stage, he always ends up with the same total energy. Why? What is that final amount of energy for 2025 stones?

§ This Issue's Contributors

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We do not claim to be the creators of any questions shared in *The Curious Quest*, unless specified otherwise.

If you have any questions, puzzles, or stories that you want to share, kindly mail them to centre.math.outreach@gmail.com!

§ Hints & Solutions - Previous Issue

Problem 1

Observe that the integrand is positive for any value of x for $e^{\cos x}$, but has positive value for the values of x in between the roots α, β where $\alpha < \beta$ of the quadratic polynomial $-x^2 - 5x + 500$ and negative value elsewhere. Argue that the maximum will be achieved for $a = \alpha$ and $b = \beta$.

Problem 2

Verify that the $g(x) > 0$ for the interval $[-22\pi, 22\pi] \implies \operatorname{sgn}(g(x)) = 1$ on the given interval. So we are ultimately solving for

$$\begin{aligned} f(x) &= 1 \\ \sin^{19} x - \cos^{26} x &= 1 \\ \sin^{19} x &= 1 + \cos^{26} x \end{aligned}$$

Since $\sin\theta \in [-1, 1]$ the RHS *must* be equal to $1 \implies \cos^{26} x = 0$, while $\sin^{19} x = 1$ simultaneously, which happens exactly once every 2π interval, and since there are 22 2π intervals in the given interval, the answer is 22.

Problem 3

Differentiate the given equation to get a linear differential equation, solve to get $f(x) = -2 + Ce^{-x^2/2}$ where $C \in \mathbb{R}$. Using the fact that $f(0) = 0$, we get $C = 2$. Thus our final function is $f(x) = 2e^{-x^2/2} - 2$. Simple analysis reveals that (d) is the only correct answer.

When Divergence met Determination

Assume the cord stretches uniformly after each tortoise move. Let step number $n \geq 1$, and let the length of the cord after n steps be $L_n = 100{,}000n$ cm. Suppose the ant is at position x_n cm along the cord after n steps.
At each step, two things happen:

1. The cord stretches from length L_n to L_{n+1}, scaling the ant's position from x_n to $\dfrac{n+1}{n}x_n$.
2. The ant then moves 1 cm forward along the new length of the cord.

This gives the recurrence:

$$x_{n+1} = \frac{n+1}{n}x_n + 1$$

Rather than solving this recurrence directly, we analyze the ant's progress as a fraction of the total cord length. Define $f_n = \dfrac{x_n}{L_n}$ as the fraction of the cord covered by the ant at step n. Since the cord is $L_n = 100{,}000n$ cm long, and the ant moves 1 cm after the stretch, the change in fractional progress per step is:

$$\Delta f_n = \frac{1}{100{,}000n}$$

Therefore,

$$f_N = \sum_{n=1}^{N} \frac{1}{100{,}000n} = \frac{1}{100{,}000} \sum_{n=1}^{N} \frac{1}{n}$$

The harmonic series $\sum \frac{1}{n}$ diverges slowly: $\sum_{n=1}^{N} \frac{1}{n} \sim \ln N + \gamma$ (Where γ is the Euler-Mascheroni constant). Thus, the fractional progress satisfies

$$f_N \sim \frac{\ln N}{100{,}000}$$

We ask when $f_N \geq 1$, meaning the ant reaches the tortoise. Solving:

$$\frac{\ln N}{100{,}000} \geq 1 \Rightarrow \ln N \geq 100{,}000 \Rightarrow N \geq e^{100{,}000}$$

This is a staggeringly large number of steps ($\approx 10^{43{,}430}$), but it is **finite**. Therefore, the ant will eventually reach the tortoise (in a universe without any end!)

Conclusion: Yes, the ant will reach the tortoise. Despite the growing distance and stretching cord, the ant's steady progress accumulates as a harmonic series, which diverges. Hence, the ant eventually traverses the entire cord.