The Curious Quest

Issue Number 6

Centre for Mathematical Outreach, MAX

"For me, mathematics is a collection of examples; a theorem is a statement about a collection of examples and the purpose of proving theorems is to classify and explain the examples."

– John B. Conway

§ Reader's Delight

Maryam Mirzakhani: The Trailblazer Who Mapped Chaos Through War and Cancer

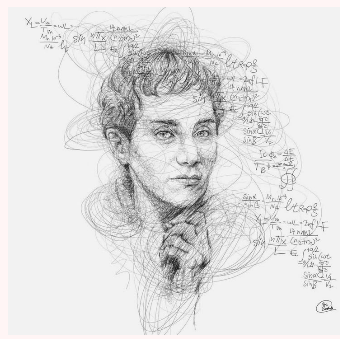


Figure 1: Maryam Mirzakhani

Maryam Mirzakhani was born in 1977 in Tehran, Iran, as bombs fell in the Iran-Iraq War. A shy girl with a taste for stories, she stumbled into math late, only to dominate it. At 17, she snagged gold at the International Math Olympiad twice; her proofs were so slick they stunned judges. Iran's universities were a slog for women, but she powered through, landing at Harvard, then Stanford, her Ph.D. in 2004 a masterstroke on hyperbolic geometry. She chased chaos curved surfaces where lines bend and billiard balls unpredictably. Her work on moduli spaces, mapping these wild terrains, was pure poetry, earning her the Fields Medal in 2014, the first woman to claim it at 37.

Life wasn't gentle. Growing up, she'd dodged war's wreckage; at Stanford, she juggled motherhood with theorems, her daughter doodling beside her desk. Then, 2013: breast cancer. She fought, sketching ideas between treatments, her husband, Jan, holding her hand. By 2017, it metastasized to her bones. At 40, she slipped away in a California hospital, Iran's flags at half-mast, mathematicians weeping worldwide. Her papers still guide topology and dynamics, her grit a torch for girls in Tehran and beyond.

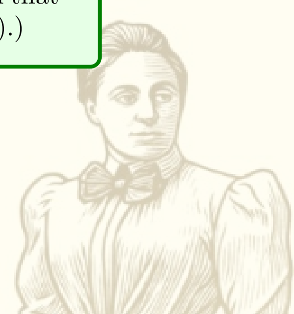
Mirzakhani mapped the unmappable, proving chaos bows to a fearless mind even one time couldn't spare.

Source: McMullen, C. (2017). "Maryam Mirzakhani's Contributions." Notices of the American Mathematical Society.

§ The Problem Arena

Problem 1

Assume the $f : \mathbb{R} \to \mathbb{R}$ is a continous, one-to-one function. If there exists a positive integer n such that $f^n(x) = x, \forall x \in \mathbb{R}$,then prove that either $f(x) = x$ or $f^2(x) = x$.(Note that $f^n(x) = f(f^{n-1}(x))$.)



Problem 2

Find the solutions for the following system of equation

$$x^3 + 9x^2y = 10$$

$$y^3 + xy^2 = 2$$

Problem 3

Three points are chosen independently and uniformly at random on the circumference of a circle with circumference 12 (i.e positions in [0,12)). Let D be the minimum distance betweeen any pair of points

1. Find the CDF of D
2. Compute $P(D > 2)$
3. Calculate $E[D]$

§ The Enigma Box

The Self-Reference Trap

Imagine a set S defined so $x \in S$ if and only if $x \notin x$ .Take S itself: if $S \in S$, then by definition $S \notin S$, but if $S \notin S$, then $S \in S$. This loops forever.S can't decide if it's a member of itself! Sounds like a glitch in reality.
Yet, set theory dodges this by saying such an S can't exist. How can a simple rule describe nothing at all? Explore this self-eating set and figure out why it's banned from the universe.

§ This Issue's Contributors

- **Problem 1**: *Aadi Upraity, St Xavier's*
- **Problem 2**: *Mark Timothy, St Xavier's*
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- **Enigma Box**: *Mark Timothy, St Xavier's*
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We do not claim to be the creators of any questions shared in *The Curious Quest*, unless specified otherwise.

If you have any questions, puzzles, or stories that you want to share, kindly mail them to centre.math.outreach@gmail.com!

§ Hints & Solutions - Previous Issue

Problem 1

$$Q(\sqrt{3}) = (a_0 + 3a_2 + 3^2a_4 + \cdots) + \sqrt{3}(a_1 + 3a_3 + 3^2a_5 \cdots) = 20 + 17\sqrt{3}$$

Since each of the a_i is equal to either 0,1 or 2, $(\cdots\ a_4\ a_2\ a_0)_3 = 20$ (Ternary expansion of 20) and similarly, $(\cdots\ a_5\ a_3\ a_1)_3 = 17$. And hence, $a_0 = 2, a_2 = 0, a_4 = 2$ and $a_1 = 2, a_3 = 2, a_5 = 1$ and all the remaining coefficients are zero. Thus,

$$Q(x) = x^5 + 2x^4 + 2x^3 + 2x + 2 \implies Q(2) = 86$$

Problem 2

Define $g(x) = f(x) - 2 \implies g(x) \in \mathbb{Z}[x]$, then,

$$g(x) = (x - a_1)(x - a_2)\cdots(x - a_{2025}) \cdot h(x)$$

For some $h(x) \in \mathbb{Z}[x]$. Suppose, for the sake of contradiction that there exists such a number $b \in \mathbb{Z}$ such that $f(b) = 9$, then $g(b) = f(b) - 2 = 9 - 2 = 7$. Now, $g(b) = 7 \implies (b - a_1)(b - a_2)\cdots(b - a_{2025}) \cdot h(b) = 7$,since it is given that all the a_i are distinct, that means that all the $(b - a_i) \in \mathbb{Z}$ are distinct, but this would imply that 7 can be written as a product of at least 2025 distinct integers, a contradiction.

Sherlock, Moriarty, and the Prison of Parity

Since any door will remain open only if it has an odd number of divisors, and only perfect squares have odd number of divisors (Prove this), we see that only the following doors remain open: 1,4,9,16,25,36,49,64,81,100