The Curious Quest

Issue Number 8

Centre for Mathematical Outreach, MAX

"Be less curious about people and more curious about ideas."

– Marie Sklodowska-Curie

§ Reader's Delight

From Verses to Vectors

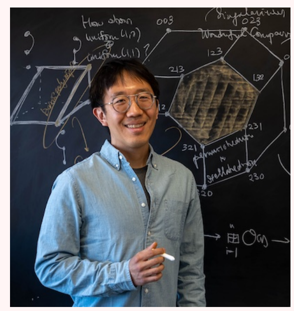

Figure 1: June Huh

It wasn't always June Huh's goal to become a mathematician. He actually dropped out of high school and spent his early years pursuing his dream of being a poet. He didn't discover algebraic geometry until he was an undergraduate, long after the majority of "math prodigies" had already won Olympiads and mastered undergraduate coursework. Even if it came late, that spark would ultimately lead him to win the 2022 Fields Medal, one of the greatest awards in mathematics.

Huh's genius resides in his extraordinary ability to combine concepts from many fields, such as abstract geometry, combinatorics, and deep algebra, to create elegant proofs that addressed persistent outstanding issues. His research on log-concavity in matroids, which was previously believed to be beyond the scope of geometric methods, stunned the mathematical community by demonstrating how approaches from complex geometry could uncover structure in discrete contexts that appeared to be unrelated. He seems to have constructed bridges connecting far-off islands and then smiled while crossing them barefoot.

June Huh's journey is a silent protest in a time where success at a young age is celebrated. It teaches us that mathematical maturity is a rhythm rather than a sprint, and those who discover their beat later frequently provide unexpected melody.

S. Roberts, "June Huh, High School Dropout, Wins the Fields Medal," *Quanta Magazine*, July 5, 2022.

§ The Problem Arena

Problem 1

In how many ways can you permute the numbers $0, 1, \dots, 9 \to \sigma(0), \sigma(1), \dots, \sigma(9)$ such that each number other than $\sigma(0)$ is within one of some number to the left of it.

Problem 2

Let $\{r_1, r_2, \ldots, r_n, ...\}$ be enumeration of all rationals in [0,1]. Define, for each $n \in \mathbb{N}$:

$$f_n(x) = \begin{cases} e^x & \text{if } x = r_1, r_2, ..., r_n \\ e^{1-x} & \text{otherwise} \end{cases}$$

Which of the following statements is true?

a) The function f_n is Riemann integrable over the interval $[0, 1]$ for each $n \in \mathbb{N}$.

b) The sequence $\{f_n\}$ is pointwise convergent and the limit function is Riemann integrable over the interval $[0, 1]$..

c) The sequence $\{f_n\}$ is pointwise convergent but the limit function is not Riemann integrable over the interval $[0, 1]$.

Problem 3

Consider the sequence given by:

$$1, 4, 5, 16, 17, ...$$

Which consists of sums of different powers of 4, that is $4^0, 4^1, 4^0 + 4^1, 4^2, ...$ in increasing order. Find out the position at which 1280 occurs.

§ The Enigma Box

Pingo

Paul shuffles a deck of cards thoroughly, then plays card face up one at a time, from the top of the deck. At any time Ringo can interrupt him and bet \$1 that the next card will be red. He bets exactly one time, if he never interrupts, he's automatically betting on the last card. What's Ringo's best strategy? How much better can he do? (Assume 26–26 red and black cards)

§ This Issue's Contributors

- **Problem 1**: *Aadi Upraity, St. Xavier's College.*
- **Problem 2**: *Aadi Upraity, St. Xavier's College.*
- **Problem 3**: *Aadi Upraity, St. Xavier's College.*
- **Enigma Box**: *Aadi Upraity, St. Xavier's College.*
- **Reader's Delight**: *Aadi Upraity, St. Xavier's College.*

We do not claim to be the creators of any questions shared in *The Curious Quest*, unless specified otherwise.

If you have any questions, puzzles, or stories that you want to share, kindly mail them to centre.math.outreach@gmail.com!

§ Hints & Solutions - Previous Issue

Problem 1

Find out all possible degrees of a_n and find out the connection between degree of a_n and p.

Problem 3

Use the so called Newton's formula to obtain a recurrence relation for γ_n and then use induction.

Flipping out

Let A be the number of heads Freddie gets from his 1991 coins, and B the number of heads Bowie gets from his 1990 coins.
Let's remove Freddie's last coin and compare his first $n = 1990$ coins to Bowie's full set. Define events:

- E_1: Freddie's first n coins have more heads than Bowie's.
- E_2: Freddie and Bowie have the same number of heads.
- E_3: Freddie's first n coins have fewer heads than Bowie's.

By symmetry, $\mathbb{P}(E_1) = \mathbb{P}(E_3) = x$, and let $\mathbb{P}(E_2) = y$. Since these are the only outcomes, we have:

$$\sum_{\omega \in \Omega} P(\omega) = 1 \implies 2x + y = 1.$$

Now consider Freddie's final (1991st) coin:

- On E_1, he already wins: contributes x to the final probability.
- On E_3, he already loses: contributes nothing.
- On E_2, his final coin is a tiebreaker:
  - Heads: Freddie wins.
  - Tails: Bowie wins.

  So E_2 contributes $\frac{1}{2}y$ to the total.

Thus, the total probability that Freddie wins i.e. $\mathbb{P}(A > B)$ is:

$$x + \frac{1}{2}y = x + \frac{1}{2}(1 - 2x) = \frac{1}{2}.$$

Answer:

$$\boxed{\mathbb{P}(A > B) = \frac{1}{2}}$$

Even when you're a bit extra like Freddie, probability plays fair.