The Curious Quest

Issue Number 13

Centre for Mathematical Outreach, MAX

"Young man, in mathematics you don't understand things. You just get used to them"

– John Von Neumann

§ Reader's Delight

The 17 year old Conquerer of The Mizohata-Takeuchi Conjecture

A paper posted on February 10 left the math world by turns stunned, delighted and ready to welcome a bold new talent into its midst. Hannah Cairo, just 17 at the time, solved a 40-year-old mystery about how functions behave, called the Mizohata-Takeuchi conjecture. The conjecture considers functions built out of waves whose frequencies satisfy equations that carve out specific surfaces, like a sphere.

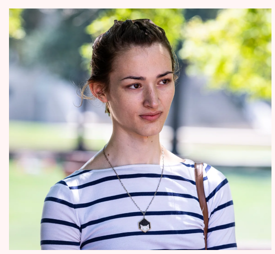

Figure 1: Hannah Cairo

Growing up in Nassau, the Bahamas, she began learning math using Khan Academy's online lessons, quickly advancing through its standard curriculum and soon moved on to graduate-level maths. She found a way to construct a strange function out of waves whose frequencies all lay on a curved surface ,the type of surface the conjecture required. Usually, the waves interfere, canceling each other out in some places and reinforcing each other elsewhere but Cairo showed that in her function, their interference created uneven patterns, causing the function's energy to spread out over some areas and concentrate in others, which the Mizohata-Takeuchi conjecture prohibited.

Hartnett, Kevin."At 17, Hannah Cairo Solved a Major Math Mystery." *Quanta Magazine*, 1 Aug. 2025, https://www.quantamagazine.org/at-17-hannah-cairo-solved-a-major-math-mystery-20250801/.

§ The Problem Arena

Problem 1

Without using L'Hosplital's rule, evaluate the following.

$$\lim_{x \to 0} \frac{x\tan(2x) - 2x\tan(x)}{(1-\cos(2x))^2}$$

Problem 2

let $\overline{abc}$ be a three digit number with non-zero digits such that

$$a^2 + b^2 = c^2$$

then what is the largest possible prime factor of $\overline{abc}$?

Problem 3

A frictionless board has the shape of an equilateral triangle of side length 1 meter with bouncing walls along the sides. A tiny super bouncy ball is fired from vertex A towards the side $\overline{BC}$. The ball bounces off the walls of the board nine times before it hits a vertex for the first time. The bounces are such that the angle of incidence equals the angle of reflection. The distance traveled by the ball (in meters) is of the form $\sqrt{N}$, where N is an integer. What is the value of N?

§ The Enigma Box

Queen Dido and the city of Carthage

Long ago, Queen Dido fled her homeland and sailed to the shores of North Africa. The local ruler offered her a curious deal:

> *"You may have as much land as you can enclose with the hide of a single ox."*

Clever Dido cut the ox hide into thin strips, tied them together into one long rope, and set out to claim her land. She stretched the rope around the sandy ground, shaping her new city's boundary.
Now imagine **you** are in her place:

- You have a fixed length of rope.
- You can lay it out however you like, forming any closed shape.
- There is no coastline to help you; the rope must completely enclose the land.
- Your goal: **capture the largest possible area**.

What shape would you choose to make the most of your precious rope?

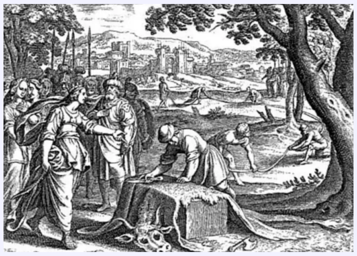

Figure 2: Queen dido with the ox hide

§ This Issue's Contributors

- **Problem 1**: *Jaden D'Cruz, St. Xavier's College*
- **Problem 2**: *Abel Alex, St. Xavier's College*
- **Problem 3**: *Ayush Mishra, St. Xavier's College*
- **Enigma Box**: *Pranit Kamath, St. Xavier's College*
- **Reader's Delight**: *Reyansh Roy, St. Xavier's College*

We do not claim to be the creators of any questions shared in *The Curious Quest*, unless specified otherwise.

If you have any questions, puzzles, or stories that you want to share, kindly mail them to centre.math.outreach@gmail.com!

§ Hints & Solutions - Previous Issue

Problem 1

Write $g(x) = x - \lfloor x \rfloor$ as $g(x) = \{x\}$ and substitute into $f(x)$ to get $f \circ g = |2\{x\} - 1| + |2\{x\} + 1|$. Plot the function for the interval $(-1, 1)$ by breaking the intervals at appropriate places. which readily reveals the number of points at which $f \circ g$ is not continuous and non-differentiable

Problem 2

$f'(x) = 1 + g'(x) > 0$ hence f is strictly increasing and injective. Also f is continuous hence it is well-known that $f : \mathbb{R} \to f(\mathbb{R})$ is a homeomorphism. For a proof: f^{-1} is strictly increasing and it is surjective, hence it cannot have jumps.

For $x \geq 0$ we have

$$f(x) = \int_0^x (1 + g'(t))\, dt \geq x(1 - k)$$

hence $f(\infty) = \infty$. In the same way $f(-\infty) = -\infty$.

Problem 3

$I = \int \sqrt[3]{\tan x}\, dx$

1. First, we let $u = \sqrt[3]{\tan x}$. This transforms the trigonometric integral into a rational function:

$$I = \int \frac{3u^3}{u^6 + 1} du$$

2. To simplify this further, we make a second substitution, $t = u^2$. Thisreduces the problem to :

$$I = \frac{3}{2} \int \frac{t}{t^3 + 1} dt$$

This final integral can now be tackled by partial fraction decomposition. The denominator $t^3 + 1$ factors into $(t + 1)(t^2 - t + 1)$,
substituting back for our original variable x, we arrive at the desired final answer:

$$\begin{aligned} I = &-\frac{1}{2} \ln\left((\tan x)^{2/3} + 1\right) + \frac{1}{4} \ln\left((\tan x)^{4/3} - (\tan x)^{2/3} + 1\right) \\ &+ \frac{\sqrt{3}}{2} \arctan\left(\frac{2(\tan x)^{2/3} - 1}{\sqrt{3}}\right) + \ C \end{aligned}$$

Technology, Speed Force, and Magic

Since $x > y > z$ then, $x \geq 3, y \geq 2, z \geq 1$. Minimum Total Points per event = 6 (3+2+1) Total points scored = 40. (22+9+9) Therefore, possible points per event: 8, 10, 20, 40 (factors of 40, with earlier minimum point requirement) Since there is more than one event, omit 40 as a possibility. Thus, number of events possible: 2, 4, 5.

2 events, 20 points each: Hence, x 8. B(The Flash) must have won 8 points and gotten +1 from another event to get a total of 9. This includes no combination where A(Iron Man) can earn 22 points. Hence, it can't be 2 events.

4 events, 10 points each: Possible points per event are (6,3,1), (5,4,1), or (5,3,2). Possibility omitted again as A can't reach 22 points.

So, it must be 5 events with a total of 8 points each, with possibilities per event as (5,2,1) or (4,3,1). A can only reach 22 points with (5,2,1), Coming first in 4 events and second in the 400m hurdles (which B won). Hence x=5, y=2, and z=1. B would need to come third in the other 4 events. Excluding the 400m hurdles, A is first in all events, and B is third in all events, implies C(Hermione) came second in all events, including the high jump event