The Curious Quest

Issue Number 14

Centre for Mathematical Outreach, MAX

"I turn with terror and horror from this lamentable scourge of continuous functions with no derivatives."

– Charles Hermite

§ Reader's Delight

Mankind's obsession with 37

Humans are surprisingly consistent when asked to pick a "random" number; across cultures and contexts, they often choose 37. This preference is tied to our perception of randomness: people tend to favour odd, prime numbers, particularly those involving 3 and 7. 37 consistently ranks among the most chosen numbers in surveys, even when respondents are asked to pick something they think no one else will pick.

Mathematically, 37 holds deeper significance. It's the median second prime factor of all integers, a unique statistical balance point. It's also central to the 37% rule in decision theory, which suggests the best strategy in one-shot decisions, like hiring, is to reject the first 37% of options, then choose the next best one, giving a 37% chance of success. All of this has led to a cultural fascination with 37, with some individuals dedicating their lives to tracking its appearances. Ironically, the more we associate 37 with randomness, the less random it becomes.

Beyond math and psychology, 37 has become a kind of cultural icon. It's a number that people notice everywhere once they're aware of it. This is partly due to confirmation bias, but also because 37 appears often enough in media, product labels, and everyday life to feel uncanny. Whether it's genuinely special or just a widely shared mental glitch, 37 has earned a place as humanity's unofficial favourite "random" number.

Veritasium: *Why is this number everywhere?* (YouTube)

§ The Problem Arena

Problem 1

Let $a \in \mathbb{R} \setminus \{-1\}$. If $\displaystyle\lim_{n\to\infty} \frac{1^a + 2^a + \cdots + n^a}{(n-1)^{a-1}[(na+1)(na+2) + \cdots + (na+n)]} = \frac{1}{60}$ then find all possible values of a.

Problem 2 ($\star$)

Find all pairs (a, b) so that each of the two polynomials $x^3 + ax + b$ and $x^3 + bx + a$ have integral roots.

Problem 3

At a rock festival, 8 bands are in the lineup: Led Zeppelin, Pink Floyd, Queen, The Beatles, The Rolling Stones, AC/DC, Metallica, and Nirvana. The organizer randomly selects 4 headline slots (in order) from these 8 bands, without repetition.
What is the probability that:

1. Exactly two of Led Zeppelin, Pink Floyd, Queen appear in the headline slots, and
2. They are not scheduled back-to-back?

§ The Enigma Box

Who scored?

To play soccer with three people, two field players try to score past the player in goal, and whoever scores stands in goal for the next game. Ada, Bernhard and Coxeter play the game, with Ada playing 12 times on the field, Bernhard playing 21 times on the field, and Coxeter playing 8 times in goal. Who scored the 6th goal?

§ This Issue's Contributors

- **Problem 1**: *Omisha Ruhi, St. Xavier's College, Mumbai*
- **Problem 2**: *Soham Mhatre, St. Xavier's College, Mumbai*
- **Problem 3**: *Martin John, St. Xavier's College, Mumbai*
- **Enigma Box**: *Omisha Ruhi, St. Xavier's College, Mumbai*
- **Reader's Delight**: *Reyansh Roy, St. Xavier's College, Mumbai*

We do not claim to be the creators of any questions shared in *The Curious Quest*, unless specified otherwise.

If you have any questions, puzzles, or stories that you want to share, kindly mail them to centre.math.outreach@gmail.com!

§ Hints & Solutions - Previous Issue

Problem 1

$$\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}=\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{4\sin^4 x}=\lim_{x\to 0}\frac{x}{4\sin^4 x}\left(\frac{2\tan x}{1-\tan^2 x}-2\tan x\right)$$
$$=\lim_{x\to 0}\frac{x}{4\sin^4 x}\left(\frac{2\tan^3 x}{1-\tan^2 x}\right)=\frac{1}{2}\lim_{x\to 0}\frac{x}{\sin x}\cdot\frac{\tan^3 x}{\sin^3 x}\cdot\frac{1}{1-\tan^2 x}=\frac{1}{2}\cdot 1\cdot 1\cdot 1=\boxed{\frac{1}{2}}$$

Problem 2

First notice $c \leq 9$ then straight forward checking of Pythagorean triplets with largest number being 9 gives the largest prime factor 29.

Problem 3

Unfold the billiards path by reflecting the triangle at each bounce. In the tiled plane of equilateral triangles, the ball travels in a straight line from A to a lattice vertex.

Let the straight-line displacement be $m\mathbf{e}_1 + n\mathbf{e}_2$, where $\mathbf{e}_1$ and $\mathbf{e}_2$ are unit vectors along two sides of the equilateral triangle with $\mathbf{e}_1 \cdot \mathbf{e}_2 = \cos 60^\circ = \frac{1}{2}$. In the *barycentric* (m, n) coordinates, lines parallel to sides AC, AB, and BC correspond to $m = \text{const}$, $n = \text{const}$, and $m + n = \text{const}$ respectively. Crossing one of these lines in the unfolding corresponds to a bounce in the original triangle. The number of crossings of these three families by the segment from $(0, 0)$ to (m, n) is

$$(m-1) + (n-1) + \big((m+n) - 1\big) \;=\; 2(m+n) - 3,$$

where we exclude the starting lines. We are told there are 9 bounces, so

$$2(m+n) - 3 = 9 \quad \Longrightarrow \quad m + n = 6.$$

For the hit at a vertex to be the *first* vertex encountered, the segment must not pass through any other lattice vertex, which is equivalent to $\gcd(m, n) = 1$. With $m, n > 0$ and $m + n = 6$, the possibilities are $(m, n) = (1, 5)$ or $(5, 1)$ (the others have $\gcd > 1$). The total distance traveled equals the Euclidean length of $m\mathbf{e}_1 + n\mathbf{e}_2$:

$$\big\|m\mathbf{e}_1 + n\mathbf{e}_2\big\|^2 = m^2 + n^2 + mn.$$

For $(m, n) = (1, 5)$ (or symmetrically $(5, 1)$),

$$N = m^2 + n^2 + mn = 1^2 + 5^2 + 1 \cdot 5 = \boxed{31}$$

Queen Dido and the city of Carthage

This puzzle wasn't shared here to be solved, but rather to spark your curiosity about a beautiful branch of mathematics known as the *Calculus of Variations*. If you'd like to see a full solution, look up "Queen Dido's Problem" — a classic that has fascinated mathematicians for centuries.