The Curious Quest

Special Edition

Centre for Mathematical Outreach, MAX

August 2025

"We must know. We will know."
– David Hilbert

§ A Small Note

Dear Reader,

Ten (plus 1) issues ago, *The Curious Quest* was just an idea, a shared dream to make mathematics feel alive, surprising, and human. Today, with this Special Edition, we pause for a moment to celebrate how far we've come.

This mini milestone is not just about numbers, it's about the curiosity we've nurtured, the questions that sparked interested in a subject, the stories that might have inspired someone, and the community we've built along the way. Each puzzle solved, each insight sparked, each smile drawn from a clever twist – they've made this journey worth every step.

Thank you for walking this path with us, whether you joined at Issue 0 or just picked this one up. Here's to many more pages of wonder ahead.

With gratitude,
The CMO Team

§ Invitation (Not for students from SXC, Mumbai)

Dear Reader,

If you enjoyed this Special Edition of *The Curious Quest*, or any of our previous 10 issues, we have just one thing to say:

Let's build more of this, together.

The **Centre for Mathematical Outreach (CMO)** is bringing together students, math clubs, and departments from across Mumbai to:

- Contribute to future editions of *The Curious Quest*
- Design collaborative lecture packs on advanced mathematical ideas
- Host intercollegiate problem-solving events and student-led math talks

Here's how you can get involved right now:

- Fill out our interest form: https://forms.gle/AkDHrqcGSoNbA8sv8
- Join our math circle group: https://chat.whatsapp.com/Cy55WBPOdIA9WKQMqF2I8h

Once we complete the formal process of reaching out to various colleges, our Networking Department will follow up with an official mail outlining:

- CMO's vision and goals for this academic year
- The role of your college as a partner in this city-wide initiative

And one last thing, the solutions to this *Special Edition* will not be uploaded online. That's intentional. We hope it becomes a spark for discussion in your college.

If you'd like to organize a group-solving session, just drop us a mail. We'd be happy to share the full solution set with one or two representatives who take the lead in hosting it.

Let's co-create a math community worth being part of.

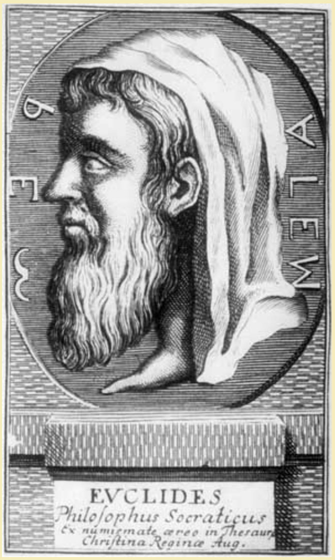
— *The CMO Team*

§ Reader's Delight: Greats throughout human history

Geometry, Number Theory & Mathematical Rigor!

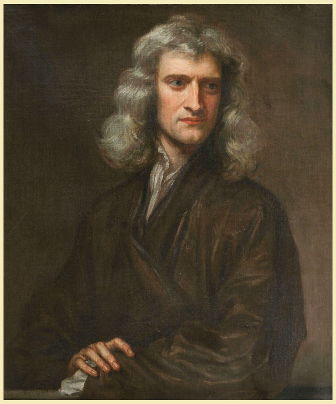
Euclid of Alexandria, thought to have lived around 300 BCE, is known for his "The Elements", a 13-part treatise of mathematics known at that time. Almost everyone associates Euclid with plane geometry, however, Geometry only took about 6 volumes of the 13 volumes of The Elements, while the other parts were devoted to Number Theory, and Solid Geometry. This is also where Euclid delivered some of his finest results, namely the Infinitude of the Primes and the Euclidean algorithm for computing the Greatest Common Divisors of two numbers.

However the defining feature of his work was the deductive nature of mathematics, setting up a handful of postulates, and deducing a plethora of results from them. The Postulate-Proposition format of his work set a standard for Pure mathematics, which we still follow to this day! (seriously, open any math textbook!)



Euclid

"I recognize the lion by its claw"



Sir Isaac Newton

The quote above, given to us by Johann Bernoulli, refers to the one of the greatest scientific mind of all time, Sir Isaac Newton. Newton was born in 1643, in a small hamlet named Woolsthorpe-by-Colsterworth. He entered Trinity College, University of Cambridge, where he obtained his BA in 1665. The university would close down soon after Newton obtained his degree, as a precaution of the Great Plague, However, The private studies Newton conducted during this "lockdown" time was to revolutionize science and mathematics to come. Newton would, On the basis of Galileo's experiments, would postulate his 3 laws of motion. Furthermore, to investigate the motion of the planets, one would invent calculus (he would call, what we call a derivative now, fluxions!) To explain the context of the quote in the title, Johann Bernoulli had solved the Brachistochrone problem, finding the path between two points at different heights, so that, say, a bead on the path would, take the least time to reach the lower point. Bernoulli gave this problem to all the leading mathematicians at the time, and had given them 6 months to solve it. Newton solved this problem in a single night! (even Leibniz asked for an extension!) To add to the mythos, had sent his solution anonymously, however to Johann it was quite obvious on who it was!

The mathematician who knew it all

Leonhard Euler

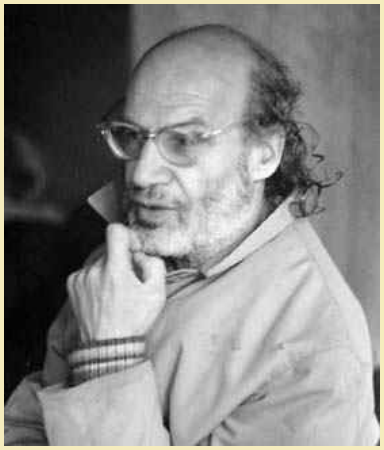
We now come to one of the greatest mathematicians to ever live; Leonhard Euler. He was born in 1707 in the town of Basel. He enrolled in the University of Basel in 1720, at the age of 14, which apparently was not uncommon for the time. He attended the mathematics lectures of Johann Bernoulli (The same one as the above!), who would not give him private lessons unfortunately, but in Euler's words himself would end up benefiting him – "but he gave me much more valuable advice to start reading more difficult mathematical books on my own and to study them as diligently as I could". Euler made numerous contribution to almost every single field of mathematics there was at the time. Even by today's standard, he is considered the most prolific mathematician of all time. To put his prolificness into perspective, Euler had discovered so many theorems and results in fields ranging from Analysis to Graph theory, that a theorem would be henceforth named after the second person who discovered that result!

One of his famous results, particularly at the time, was the solution to the Basel problem, where he showed that the sum

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$$

Another one of his famous results, which is regarded by many as the "The most beautiful equation", is the relation $e^{i\theta} = cos(\theta) + isin(\theta)$, which can be roughly thought of as encoding the rotation of a vector in the complex plane by a angle of θ, it is the special case of this relation where $\theta = \pi$ that gives us the formula $e^{i\pi} = -1$. We end this reader's delight with another result discovered by Euler, that gives us the relation between the vertices, edges and faces of a convex polyhedra $V - E + F = 2$.

The Greatest of the 20th Century



Alexander Grothendieck

To end this reader's delight, and our journey of mathematicians of the ages, we now get to Alexander Grothendieck, who's widely considered to be one of the greats in contemporary mathematics. Born in 1928 in Berlin into a Jewish family, they had to go through severe hardships due to the ongoing tensions in Europe, which would lead to WW2. Grothendieck and his mother would move to France in 1939 as the war broke out and would be interned into camps throughout the remainder of the war. After the war, He would then go on to study mathematics at University of Montpellier, however he was unhappy with the instruction there, so much so he would try to 'rebuild' mathematics from the ground up, on his own.

At the IHES, he would then go on to make substantial contributions to Functional Analysis, Algebraic Geometry. His work over there saw a unification between Number Theory, Geometry, Topology and Complex Analysis , for which he would be awarded the Fields Medal in 1966. His Fields Medal citation reads – "Grothendieck built on work of Weil and Zariski and effected fundamental advances in algebraic topology. He introduced the idea of K-theory (the Grothendieck groups and rings). He revolutionized homological algebra in his celebrated 'Tohoko paper'." He would eventually live in seclusion after his retirement, in a small village in the Pyrenees. He passed away at the age of 86 in 2014.

§ Sources

- O'Connor, J. J., & Robertson, E. F. (1999). *Euclid.* MacTutor History of Mathematics Archive. University of St Andrews. https://mathshistory.st-andrews.ac.uk/Biographies/Euclid/
- O'Connor, J. J., & Robertson, E. F. (1996). *Isaac Newton.* MacTutor History of Mathematics Archive. University of St Andrews. https://mathshistory.st-andrews.ac.uk/Biographies/Newton/
- Cantor's Paradise. (2019, June 10). *The famous problem of the Brachistochrone.* https://www.cantorsparadise.org/the-famous-problem-of-the-brachistochrone-8b955d24bdf7/
- O'Connor, J. J., & Robertson, E. F. (1998). *Leonhard Euler.* MacTutor History of Mathematics Archive. University of St Andrews. https://mathshistory.st-andrews.ac.uk/Biographies/Euler/
- O'Connor, J. J., & Robertson, E. F. (2005). *Alexander Grothendieck.* MacTutor History of Mathematics Archive. University of St Andrews. https://mathshistory.st-andrews.ac.uk/Biographies/Grothendieck/

§ The Problem Arena

A small note:
While some of these problems may have solutions available online, we strongly encourage you to work through them with your college's mathematics club. Solving these puzzles collaboratively—with your peers, juniors, and seniors—not only deepens your understanding, but also fosters a culture of shared curiosity and discovery.

Problem 1 — Elementary Calculus

For $n \geq 1$, define $\mathscr{C}^n(I)$ to be the set of all functions on I such that it is differentiable n times (or ONLY continuous for $n = 0$). Consider two functions $f, g \in \mathscr{C}^1[a-1, a+1]$ and If $f(a) = 2, f'(a) = 1, g(a) = -1$ and $g'(a) = 2$, then find the value of:

$$\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x - a}$$

Problem 2 — Elementary Number Theory

What is the least positive integer by which $2^5 \cdot 3^6 \cdot 4^3 \cdot 5^3 \cdot 6^7$ should be multiplied so that, the product is a perfect square ?

Problem 3 — What appears is not what happens

Let $\{a_n\}$ be a sequence of real number. Suppose that $\{a_{sn}\}$ converges for every $s > 1$.

1. If $a_{sn} \to a$ and $a_{tn} \to b$ for some fixed positive integers s and t then is $a = b$? Justify.
2. Is the sequence a_n convergent? Justify.

Problem 4 — Hungry?

Let $f \in \mathscr{C}^0([0,1])$. Calculate:

$$\lim_{n \to \infty} \int_0^1 x^n f(x)\, dx$$

Problem 5 — Numeric Bounding

A 5-digit number (in base 10) has digits $k, k+1, k+2, 3k, k+3$ in that order, from left to right. If this number is m^2 for some natural number m, find m.

Problem 6 — Hulk

Consider the set $E = \{5, 6, 7, 8, 9\}$. For any partition $\{A, B\}$ of E, with both A and B nonempty, consider the number obtained by adding the product of elements of A to the product of elements of B. Let N be the largest prime number among these numbers. Find N.

Problem 7 — Straight from inspiration

(a) Let $f \in \mathscr{C}^2(\mathbb{R})$, show that:

$$\lim_{h\to 0} \frac{f(x+h)+f(x-h)-2f(x)}{h^2} = f''(x)$$

for all $x \in \mathbb{R}$.
(b) Show that if f also satisfies:

$$\frac{1}{2y}\int_{x-y}^{x+y} f(t)\,dt = f(x)$$

then f is linear.

Problem 8 — Rooted in Mystery

The product of two of the four roots of the quartic equation $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ is -32. Determine the value of k.

Problem 9 — Divisors

Find all pairs of positive integers α, β such that $\alpha\beta \mid \alpha^{2025} + \beta$.

Problem 10 — Stargazing

Let $\{x_n\}_{n\geq 0}$ be the sequence such that $x_0 = 1$ and for $n \geq 0$,

$$x_{n+1} = \ln(e^{x_n} - x_n)$$

Show that the infinite series

$$x_0 + x_1 + x_2 + \cdots$$

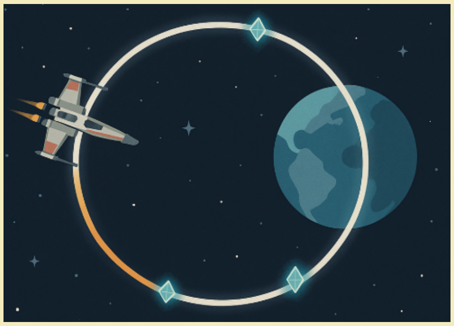
converges and find its sum.

Problem 11 — Cute analysis

Consider a sequence $\{r_k\}_{k\geq 1}$ of rational numbers lying in the interval $(0,1)$. Further, assume that r_k converges to an irrational number as $k \to \infty$. Suppose $r_k = \frac{m_k}{n_k}$, for all $k \geq 1$, where m_k and n_k are positive integers with no common divisors. Show that the set of integers $\{n_k : k \geq 1\}$ is not bounded.

§ The Enigma Box

The Hyperspace Loop Paradox



Can the Rebellion count on your navigation?

You're piloting a **Rebel starfighter** on a one-way hyperspace ring encircling a planet occupied by the Empire. The total circumference of the hyperspace ring is normalized to 1 unit. Scattered across this ring are N **Kyber fuel cells**, placed at fixed positions. The sum of all their fuel is exactly enough to complete one full loop around the ring. However, your starfighter starts with **no fuel** onboard. You are allowed to start at any one of the fuel cells, collecting it instantly when you pass. Your engine consumes fuel at a constant rate proportional to the distance traveled. Can you always choose a starting point such that your starfighter can complete the full hyperspace loop, no matter where the Kyber fuel cells are placed?

The Weekday Whopper Conundrum

Let S be a person who lies on exactly six days of the week and tells the truth on exactly one day (the same day each week).

On three consecutive calendar days — referred to as **Day 1**, **Day 2**, and **Day 3** — he makes the following statements:

- **Day 1**: "I lie on Mondays and Tuesdays."
- **Day 2**: "Today is Thursday, Saturday, or Sunday."
- **Day 3**: "I lie on Wednesdays and Fridays."

Assume each statement was made on the actual day it refers to (i.e., the person made the "Day 1" statement on some day of the week, and similarly for "Day 2" and "Day 3"). Determine the unique day of the week on which S tells the truth.

The Enchanted Square of Lies

The Order of Arithmancers has conjured a legendary 3×3 **Magic Square**, where all rows, columns, and diagonals must sum to 15, using the numbers 1 through 9 exactly once.

But a mischief spell has scrambled the square, and worse, 8 of the 9 wizards guarding each cell have been cursed to lie. Only **one wizard speaks the truth**.

Each wizard stands before a cell in the grid and announces what number they claim lies within. But only one claim is truthful. The rest? Pure magical misdirection.

Below are the statements made by the nine wizards, standing in a 3×3 grid layout. Each says:

The Order

(1,1): "This cell contains a **5**."	(1,2): "This cell contains a **1**."	(1,3): "This cell contains a **9**."
(2,1): "This cell contains a **6**."	(2,2): "This cell contains a **7**."	(2,3): "This cell contains a **2**."
(3,1): "This cell contains a **4**."	(3,2): "This cell contains a **3**."	(3,3): "This cell contains an **8**."

Can you deduce which wizard tells the truth and reconstruct the true Magic Square?

And we are done!

Thank you for reading.