The Curious Quest

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Centre for Mathematical Outreach, MAX

"Read Euler, read Euler, he is the master of us all"

– Pierre-Simon Laplace

§ Reader's Delight

The mathematician who came out of nowhere

The story of George Green is a remarkable one, not least because he began life far removed from the halls of academia. Born in 1793 in Sneinton, a small village near Nottingham, Green worked in his father's bakery and later the family windmill. He received only a modest schooling, yet his passion for mathematics burned bright. In 1828, at the age of 35, Green privately published his Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism. This obscure little pamphlet would later prove revolutionary: it contained what we now call Green's Theorem, a cornerstone of modern vector calculus.

Figure 1: George Green

What is striking is that Green had essentially no access to books or scholarly networks, relying instead on perseverance and self-study. His essay was so far ahead of its time that it went largely unnoticed, until William Thomson (later Lord Kelvin) rediscovered it years later. Thomson was astonished, saying Green's work "astonished mathematicians by its profound originality." After inheriting the mill, Green finally enrolled at Cambridge in 1833, entering Gonville and Caius College at the age of forty. Sadly, his career there was short as he died in 1841 at only 47 years old. Yet in his brief life, George Green left behind ideas that continue to echo throughout mathematics and physics to this day.

O'Connor, JJ., and E.F. Robertson. "George Green." MacTutor History of Mathematics Archive, University of St Andrews. https://mathshistory.st-andrews.ac.uk/Biographies/Green/.

§ The Problem Arena

Problem 1

Let function f satisfy

$$xf(x) = ln(x), \text{ for } x > 0$$

Show that,

$$f^{(n)}(1) = (-1)^{n+1}n!(1 + \frac{1}{2} + \cdots + \frac{1}{n})$$

where $f^{(n)}$ is the nth derivative of the function f evaluated at x

Problem 2

A fair die is rolled 3 times. The conditional probability of 6 appearing exactly once, given that it appeared at least once already, is

a) $\frac{3[1/6]^2[5/6]}{1-[5/6]^3}$

b) $\frac{[1/6][5/6]^2}{1-[5/6]^3}$

c) $\frac{3[1/6][5/6]^2}{1-[5/6]^3}$

d) $\frac{[1/6]^2[5/6]}{1-[5/6]^3}$

Problem 3

Find the sum of the first 50 terms of the sequence:

$$S = \frac{2}{1^2+3^2} + \frac{3}{1^2+3^2+5^2} + \frac{4}{1^2+3^2+5^2+7^2} + \cdots + \frac{51}{\sum_{k=1}^{50}(2k-1)^2}$$

§ The Enigma Box

The Postman Workout

A postman has deliveries to make on a long street, to addresses $2, 3, 5, 7, 11, 13, 17,$ and 19. The distance between any two houses is proportional to the *difference* of their addresses.

To minimize the distance traveled, the postman would of course make his deliveries in **increasing** (or **decreasing**) order of address. But our postman is overweight and would like to *maximize* the distance traveled making these deliveries, so as to get the most exercise he can. But he can't just wander around town; to do his job properly he is obligated to walk directly from each delivery to the next one. In what order should he make his deliveries?

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> We do not claim to be the creators of any questions shared in *The Curious Quest*, unless specified otherwise.

If you have any questions, puzzles, or stories that you want to share, kindly mail them to centre.math.outreach@gmail.com!

§ Hints & Solutions - Previous Issue

Problem 1

$$\lim_{n\to\infty} \frac{1^a + 2^a + \cdots + n^a}{n^{a-1}(1+\frac{1}{n})^{a-1}(n[(a+\frac{1}{n}) + (a+\frac{2}{n}) + \cdots + (a+\frac{n}{n})])}$$

Multiplying and dividing by $\frac{1}{n}$ and simplifying we obtain

$$\frac{\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}(\frac{r}{n})^a}{\lim_{n\to\infty}\frac{1}{n}\sum r = 1^n(a+\frac{r}{n})}$$

Then use the limit sum property of integration to obtain

$$\frac{\int_0^1 x^a dx}{\int_0^1 (a+x)dx} = \frac{2}{(a+1)(2a+1)} = \frac{1}{60}$$

Solve for a to get

$$a = 7, \ \frac{-17}{2}$$

Problem 2

You will need to use the following lemma

Lemma 1. *if α, β, γ are reals so that $\alpha + \beta + \gamma = 0$ and $|\alpha|, |\beta|, |\gamma| \geq 2$, then*

$$|\alpha\beta + \beta\gamma + \gamma\alpha| < |\alpha\beta\gamma|$$

using the above and Vieta's relation, you can show, by contradiction, that at least one of the roots has absolute value of less than 2, i.e one of the roots is $\{-1, 0, 1\}$. Then assume WLOG a root for each of the 3 cases. This will lead to the conclusion that $(0,0)$ is the only solution pair

Problem 3

The total number of ordered choices is

$$P(8,4) = 8 \times 7 \times 6 \times 5 = 1680.$$

Step 1: Choose exactly 2 bands from $\{LZ, PF, Q\}$: $\binom{3}{2} = 3$.

Step 2: Choose 2 more from the remaining 5 bands: $\binom{5}{2} = 10$.
So the total number of chosen sets is

$$3 \times 10 = 30.$$

Step 3: Arrange these 4 bands. The total number of possible orders is

$$4! = 24.$$

Among these, the orders where the two special bands are adjacent are

$$3! \times 2! = 12.$$

Therefore, the valid orders are

$$24 - 12 = 12.$$

Favourable outcomes:

$$30 \times 12 = 360.$$

Probability:

$$\frac{360}{1680} = \frac{3}{14}.$$

Who scored?

Whenever one player is in goal, the other two players must be on the field. So the number of times Ada played in the field must equal the number of times Bernhard played in goal plus the number of times C played in goal. Coxeter played in goal 8 times, and Ada played on the field 12 times, so Bernhard must have been in goal 4 times.

Similarly, since Bernhard played on the field 21 times and Coxeter played in goal 8 times, Ada must have played in goal 13 times. So the number of times Ada, Bernhard and Coxeter played in goal was 13, 4 and 8 respectively.

This means Ada was in goal for 13 of the 25 games, just over half the time. But due to the rule about players swapping in and out, no player can play in goal during successive games. The only conclusion is that Ada played in goal for all the odd numbered games: game 1, 3, 5 and so on, up to game 25. In particular, Ada was in goal for the 7th game, and thus it was Ada who scored the 6th goal.